Collision Detection trough Deconstruction of Articulated Objects*

Roberto Therón, Vidal Moreno, Belén Curto and Francisco J. Blanco

Departamento de Informática y Automática
Universidad de Salamanca, Salamanca, 37008, Spain
theron@usal.es, vmoreno@abedul.usal.es
bcurto@abedul.usal.es, jblanco@abedul.usal.es

Abstract. Many applications in computer graphics require fast and robust collision detection algorithms. The problem of simulating motion in an articulated chain has been well studied using both dynamic and kinematics techniques. This paper describes an efficient method for obstacle representation in the configuration space (C-space) for articulated chains. The method is based on the analytical deconstruction of the C-space, i.e., the separated evaluation of the C-space portion contributed by the collisions of each link. The Deconstruction method is not limited to particular kinematic topologies and allows good collision detection times. The systematic application of a simple convolution of two functions describing each link in the kinematic chain and the workspace, respectively, is applied. The proposed method can naturally face the evaluation of high-dimensional C-spaces, since only non-colliding configurations are considered for the evaluation of the next link in the chain.

Key words: collision detection, interference tests, motion planning

1 Introduction

This paper presents a novel and efficient method for the evaluation of possible collisions of any articulated body in an environment of obstacles.

Collision detection is a classical problem in computer graphics, robotics, manufacturing, animation and computer simulated environments. The goal of collision detection (also known as interference detection or contact determination) is to automatically report a geometric contact when is about to occur or has actually occurred. In many of these application areas, collision detection is considered a major computational bottleneck. This problem has been widely studied; [1],[2] and [3] provide recent surveys.

The motion of articulated bodies has been a subject of considerable literature using both dynamic and kinematics techniques. While inverse kinematics models are computationally less expensive, dynamics models achieve a greater degree of

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realism due to underlying physical basis. On the other hand, inverse kinematics, owing to kinematic constrains, enable more direct animations than in purely dynamic models. Usually the emphasis is on simulating an articulated figure as realistic as possible. Although realism is a worthy goal, designing interactive environments requires efficient performance [4]. With this goal in mind, Bandi and Thalmann [4] propose a configuration space approach for efficient animation of human figures, where the configuration space is splitted into various regions, mapped onto 2D, and a search is carried out to avoid obstacles. [5][6] also perform a configuration space search in order to achieve collision avoidance.

The concept of configuration space was introduced by Lozano-Pérez [7] and has been widely used in motion planning. The goal of motion planning is to generate a collision-free path for a robot. Thus, collision-free planners must be able to perform some kind of geometric reasoning concerning collision detection between the robot and the obstacles [8]. In general, the configuration of a robot is given by a set of parameters, or degrees of freedom, that determine its location and orientation. The space defined by the ranges of allowed values for these parameters is usually called configuration space (C-space).

An obstacle in C-space (C-obstacle) is defined as the connected set of configurations where a given mobile object intersects with an obstacle in workspace. C-obstacle generation can be viewed as a further generalization of the static interference and collision detection problems: here objects are not tested for interference at a particular configuration nor even along a given parameterized trajectory, but rather at all possible configurations in the workspace. Thus, once C-obstacles are obtained, all information concerning interferences is captured[1].

Concisely, we propose a fast method for the evaluation of the configuration space of articulated bodies (kinematic chains) based on the analytical deconstruction of the C-obstacles [9], that can be further exploited in computer graphics and animation. Some benefits of the proposed method are: it is valid for any kind of structure (including highly articulated bodies), the evaluation of obstacles is performed locally for every element of the articulated chain, the anticipation of collisions due to each link permits to diminish the portion of evaluated space and the method is inherently parallel.

2 Evaluating C-Obstacles as a Convolution

In this section, the method proposed by Curto et al. in [10] is reviewed, as it is the basis for the method presented in this paper.

The representation of the C-obstacles is proposed based on the integral of the product of two functions: one that represents the kinematic chain A and another one that represents the obstacles in the workspace, B. W will designate the workspace and C the C-space. Thus,

Definition 1. *Let $A : C \times W \rightarrow R$ be the function defined by*

$$A(q,x) = \begin{cases} 1 \text{ } \textit{if } x \in \mathbf{A}(q) \\ 0 \text{ } \textit{if } x \notin \mathbf{A}(q) \end{cases} \tag{1}$$

where $\mathbf{A}(q)$ *is the subset of* W *that represents the chain at configuration* q*.*

Definition 2. *Let* $B : W \rightarrow R$ *be the function defined by*

$$B(x) = \begin{cases} 1 \text{ } \mathit{if} \text{ } x \in \mathbf{B} \\ 0 \text{ } \mathit{if} \text{ } x \notin \mathbf{B} \end{cases} \tag{2}$$

where $\mathbf{B}$ *is the subset of* W *formed by the obstacles.*

Using both A and B, a new definition for calculating C-obstacles is proposed:

Definition 3. *Let* $CB : C \rightarrow R$ *be the function defined by*

$$CB(q) = \int A(q,x)B(x)dx \quad \forall q \in C, \quad \forall x \in W \tag{3}$$

The region $\mathrm{CB}_{\mathbf{f}}$ *is defined as the subset of* C *that verifies*

$$\mathbf{C}\mathrm{B}_{\mathbf{f}} = \{q \in C / CB(q) > 0\} \tag{4}$$

The previous expressions were defined without considering any specific parameterization of W and C.

Now, a representation of W and C is given by selecting two frames F_W and F_A for the workspace and for the kinematic chain, respectively, where F_W is fixed and F_A is attached to the kinematic chain. In this way, a point $x \in W$ is given by $(x_1, x_2, \cdots, x_n)$ where n is the workspace dimension, and a configuration $q \in C$ is represented by $(q_1, q_2, \cdots, q_m)$ that specify the position and orientation of F_A respect to F_W, where m is the dimension of C. Thus, the expression (3) becomes

$$CB(q_1, \cdots, q_m) = \int A(q_1, \cdots, q_m, x_1, \cdots, x_n)B(x_1, \cdots, x_n)dx_1 \cdots dx_n \tag{5}$$

3 Superposition Principle of C-obstacles

In this paper, an articulated body is considered as a kinematic chain. In this way, a body $\mathbf{A}$ is viewed as a set of r rigid objects. The kinematics of this chain, i.e., the movement restrictions imposed by the joint to each element, $\mathbf{A}_i$ —the degrees of freedom, DOFs—, would determine some regions of the C-space.

This principle is the basis of the evaluation of the C-space for bodies that consist of several links connected by means of different types of joints.

Considering that a body consists of r rigid objects, the resulting C-obstacles will follow the Superposition Principle:

Theorem 1. *Let* $\mathbf{A}$ *be an articulated body formed by* r *links* $\mathbf{A}_1, \ldots, \mathbf{A}_r$*. If* $\mathbf{CB}_1, \ldots, \mathbf{CB}_r$ *are, respectively, the C-obstacle regions for the* $\mathbf{A}_1, \ldots, \mathbf{A}_r$ *objects in the space where the obstacle* $\mathbf{B}$ *is projected, then, the C-obstacle* $\mathbf{CB}$ *due to* $\mathbf{B}$ *for the articulated body* $\mathbf{A}$ *can be obtained as*

$$\mathbf{CB} = \bigcup_{k=1}^{r} \mathbf{CB}_k \tag{6}$$

The expression (6) reflects the fact that the union of these subsets equals the configuration space for **A**. The idea of C-obstacles superposition is the key principle that enables the deconstruction approach.

4 The Deconstruction Method

The Deconstruction method tries to independently evaluate portions of the C-space in order to find the C-obstacles due to each link in the kinematic chain.

4.1 Applying the Superposition Principle

Taking into account (6), the calculation of **CB** for a body **A**, a kinematic chain of r links, is done through the union of all the $\mathbf{CB}_k$ related to each of the links of the chain. The computation of every C-obstacle region must be done through the evaluation of the associated CB_k functions.

$$CB_k(q_{1_k}, \cdots, q_{s_k}),\ \forall k \in \{1, \ldots, r\} \tag{7}$$

with $\{q_{1_k}, \cdots, q_{s_k}\} \subseteq \{q_1, \cdots, q_m\}$, where $\{q_1, \cdots, q_m\}$ are the DOFs associated to the articulated body **A**. That is, for the *k-th* element only the subset of configuration variables associated to it are considered, and, analogously to (5), each of the $CB_k(q_{1_k}, \cdots, q_{s_k})$ functions is evaluated as follows

$$\int A_k(q_{1_k}, \cdots, q_{s_k}, x_1, \cdots, x_n) B(x_1, \cdots, x_n) dx_1 \cdots dx_n \tag{8}$$

4.2 Choosing the Frames

When solving the integral (8), the function $A_k(q_{1_k}, \cdots, q_{s_k}, x_1, \cdots, x_n)$, representing the articulated body, is difficult to evaluate, due to its dependency on all of the DOFs related to itself and to the previous links in the chain. Thus, we will try to reduce this difficulty by choosing the proper frames.

In order to do that, let's consider the body formed by the kinematic chain of figure 1. As one can see, following the Denavit-Hartenberg method [11], a frame is associated with each link, placing the origin at the end of the link; the orientation of axes depends on the position and orientation of the link.

Following the Denavit-Hartenberg procedure, the Deconstruction method proposes to use the frame determined by the previous link for the *k-th* element. Thus, for link 1 the frame F_{A_0} —which coincides with the workspace frame, F_W— is used; similarly, for the *k-th* link, frame $F_{A_{k-1}}$ will be used (figure 1).

Now, if we have a look to $A_k(q_{1_k}, \cdots, q_{s_k}, x_1, \cdots, x_n)$, the expression we are evaluating, it can be written as follows

$$A_k(\underbrace{q_{1_k}, \cdots, q_{u_k}}_{DOF_{(1,\ldots,k-1)}}, \underbrace{q_{(u+1)_k}, \cdots, q_{s_k}}_{DOF_k}, x_1, \cdots, x_n) \tag{9}$$

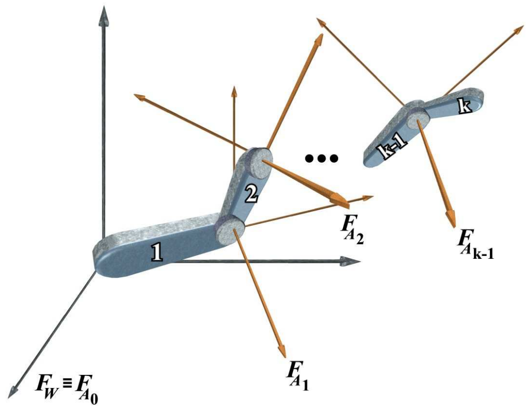

Fig. 1. Frames in the kinematic chain of an articulated body

where $\{q_{1_k}, \cdots, q_{u_k}\}$ are the degrees of freedom associated to the elements preceding the k-th element, whose DOFs are $\left\{q_{(u+1)_k}, \cdots, q_{s_k}\right\}$.

At this point, the position and orientation of the element $\mathbf{A}_k$ is expressed related to the frame F_{A_0}. The position, just like the frame $F_{A_{k-1}}$, is determined by the associated degrees of freedom of the previous links in the chain, that is to say, some of the parameters related to each $\mathbf{A}_i$ —previous elements— in that subchain, (a_i, α_i, d_i and θ_i, the Denavit-Hartenberg parameters).

Thus, if the position and orientation of the element $\mathbf{A}_k$ are expressed taking as origin the frame $F_{A_{k-1}}$, its evaluation will be much simpler. An homogeneous transformation $\mathbf{T}$ is needed to perform this operation.

Definition 4. *Let* ${}_0^{k-1}\mathbf{T}$ *be the transformation that permits to move the frame* F_{A_0} *to such point that it will coincide with* $F_{A_{k-1}}$.

It is important to point out that this homogeneous transformation depends on the configuration parameters related to the previous elements in the chain, that is to say, ${}_0^{k-1}\mathbf{T} = f(q_{1_k}, \cdots, q_{u_k})$. At this point, the position and orientation of the link $\mathbf{A}_k$, expressed related to the frame $F_{A_{k-1}}$, will only depend on its associated degrees of freedom, that is, $\left\{q_{(u+1)_k}, \cdots, q_{s_k}\right\}$.

However, this homogeneous transformation has a consequence: it will be necessary to express the workspace as a function of the new frame, $F_{A_{k-1}}$:

$$B'(x'_1, \cdots, x'_n) = {}_0^{k-1}\mathbf{T}B(x_1, \cdots, x_n) \tag{10}$$

In this way, the evaluation of (9) is equivalent to the following one

$$A'_k(q_{(u+1)_k}, \cdots, q_{s_k}, x'_1, \cdots, x'_n) \tag{11}$$

Finally, (8), which is used to calculate the C-obstacle portion pertaining to the element $\mathbf{A}_k$, becomes

$$\int A'_k(q_{(u+1)_k}, \cdots, q_{s_k}, x'_1, \cdots, x'_n) B'(x'_1, \cdots, x'_n) dx'_1 \cdots dx'_n \quad (12)$$

Now, after the proper frame is chosen, as it can be seen in (12), it is possible to study individually each one of the links.

4.3 Choosing the Coordinate Functions

Kavraki [12] and Curto [10] propose the simplification of the C-space calculation by using of the Convolution theorem (and the Fast Fourieer Transform). We shall now expose how this is applicable inside the new proposed formalism by means of the introduction of a coordinate functions change.

As demonstrated in [10], it is sufficient to choose the proper coordinate functions, $(\xi_1, \cdots, \xi_n)$, that will permit to find one or more relationships between some of the configuration variables and some of the coordinate functions, which will allow to find the convolution.

Thus, a new function, $\bar{A}'_k$, is introduced; the idea is to find a simpler functional dependency in function A'_k, in such a way that element $\mathbf{A}_k$ becomes independent of a subset of $\{q_{(u+1)_k}, \cdots, q_{s_k}\}$, depending only on $\{q_{(v+1)_k}, \cdots, q_{s_k}\}$.

Having this new function $\bar{A}'_k$, (12) will be defined as

$$\int longA \; longB \; d\xi_1 \cdots d\xi_n$$

$$\begin{array}{l} longA = \bar{A}'_k(0, \cdots, 0, q_{(v+1)_k}, \cdots, q_{s_k}, \xi_1 - q_{(u+1)_k}, \cdots, \xi_v - q_{v_k}, \xi_{(v+1)_k}, \cdots, \xi_n) \\ longB = B'(\xi_1, \cdots, \xi_n) \end{array} \quad (13)$$

which leads to a function $\bar{A}'_k$ that depends only on $\{q_{(v+1)_k}, \cdots, q_{s_k}\}$. Now, for variables $\{q_{(u+1)_k}, \cdots, q_{v_k}\}$ the following convolution product appears.

$$\int (\bar{A}'_{k(0,\cdots,0,q_{(v+1)_k},\cdots,q_{s_k})} * B)_{(\xi_1,\cdots,\xi_{v_k})} (\xi_{(v+1)_k}, \cdots, \xi_n) \, d\xi_{(v+1)_k} \cdots d\xi_n \quad (14)$$

where subindices $(\xi_1, \cdots, \xi_{v_k})$ denote that the convolution product is calculated for all of the values of these variables.

5 Case study: deconstruction of an arm in 3D

For simplicity's sake a simple example of a 3-DOF arm is considered.

Let's consider the following articulated arm, $\mathbf{A}$, consisting of 3 rigid objects, $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$, moving in R^3 by means of revolution joints. The three DOF are $(\theta_1, \theta_2, \theta_3) \in [-\pi, \pi)$. Being the waist (θ_1), shoulder (θ_2) and elbow (θ_3).

Choosing the Frames The frames are chosen following the Denavit-Hartenberg method, with the objective of obtaining certain symmetries that will simplify the calculation of the C-obstacles. F_W and F_{A_0} frames have their origins located at the intersection point of the two elements $\mathbf{A}_1$ and $\mathbf{A}_2$.

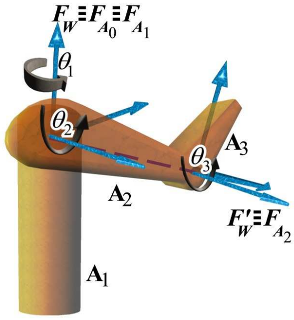

Fig. 2. A 3-DOF arm in 3D workspace

Choosing the Coordinate Functions and CB calculation Together with the frame choosing step, this will produce great simplification in the evaluation.

Indeed, the degrees of freedom associated to the second element are the two turning angles in the three-dimensional space, so the election of spherical coordinates $(r, \varphi_1, \varphi_2)$ within $[0, l_2 + l_3] \times [-\pi, \pi) \times \left[\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (with l_2 and l_3, the longitudes of the second and third element, respectively) is the best option, since θ_1 will be related to φ_1 and θ_2 to φ_2.

Following the deconstruction idea, we want to solve separately the group of collisions associated to each one of the three links of the chain.

$$\mathbf{CB} = \mathbf{CB}_1 \cup \mathbf{CB}_2 \cup \mathbf{CB}_3 \tag{15}$$

this way, three functions (expression 7) must be evaluated, and, according to expression 8, this can be done as follows

$$CB_1(\theta_1) = \int A_1(\theta_1, r, \varphi_1, \varphi_2) B(r, \varphi_1, \varphi_2) dr d\varphi_1 d\varphi_2 \tag{16}$$

$$CB_2(\theta_1, \theta_2) = \int A_2(\theta_1, \theta_2, r, \varphi_1, \varphi_2) B(r, \varphi_1, \varphi_2) dr d\varphi_1 d\varphi_2 \tag{17}$$

$$CB_3(\theta_1, \theta_2, \theta_3) = \int A_3(\theta_1, \theta_2, \theta_3, r, \varphi_1, \varphi_2) B(r, \varphi_1, \varphi_2) dr d\varphi_1 d\varphi_2 \tag{18}$$

where functions A_k and B of the formalism proposed in section 4 are parameterized for this case as $q = (\theta_1, \theta_2, \theta_3)$ and $x = (r, \varphi_1, \varphi_2)$.

At this point, we have considered that element $\mathbf{A_1}$, the waist, is only responsible of another degree of freedom for the shoulder, but we are only interested in the collisions of the arm. This way, expression 16 is null; in other case it should be computed. The evaluation of other CB follows.

Use of Convolution for CB_2 calculation In the first place, the relationships of φ_1 with θ_1 and φ_2 with θ_2 are important, since we can introduce the following expression.

$$A_2(\theta_1, \theta_2, r, \varphi_1, \varphi_2) = A_2(0, 0, r, \varphi_1 - \theta_1, \varphi_2 - \theta_2) \tag{19}$$

and changing the notation for element $\mathbf{A}_2$ at *zero* configuration ($\theta_1 = 0, \theta_2 = 0$) we have

$$A_2(0, 0, r, \varphi_1 - \theta_1, \varphi_2 - \theta_2) = A_{2_{(0,0)}}(r, \varphi_1 - \theta_1, \varphi_2 - \theta_2) \tag{20}$$

With this simple change an enormous advantage is gained, since the evaluation of the function A_2 is reduced to considering the element at configuration ($\theta_1 = 0, \theta_2 = 0$), instead of evaluating for each value $\theta_1, \theta_2 \in [-\pi, \pi)$.

So $CB_2(\theta_1, \theta_2)$ calculation is carried out by the following integral

$$\int A_{2_{(0,0)}}(r, \varphi_1 - \theta_1, \varphi_2 - \theta_2) B(r, \varphi_1, \varphi_2) dr d\varphi_1 d\varphi_2 \tag{21}$$

And, considering the convolution of both functions defined in R^3 over the θ_1 and θ_2 variables, it is obtained

$$CB_2(\theta_1, \theta_2) = \int (\bar{A}_{2_{(0,0)}} * B)_{(\varphi_1, \varphi_2)}(r, \theta_1, \theta_2) dr \tag{22}$$

where subindex (φ_1, φ_2) means that the convolution product of functions $\bar{A}_2$ and B is carried out for all the values of variables $(\varphi_1, \varphi_2) \in [-\pi, \pi)$, and function $\bar{A}_{2_{(0,0)}}$ is defined by

$$\bar{A}_{2_{(0,0)}}(r, \varphi_1, \varphi_2) = A_{2_{(0,0)}}(r, -\varphi_1, -\varphi_2) \tag{23}$$

Finally, the convolution theorem can be applied, so now the expression 22 is calculated with the inverse Fourier transform (over two dimensions) of

$$\int \mathcal{F}\left[\bar{A}_{1_{(0,0)}}(r, \theta_1, \theta_2)\right]_{(\varphi_1, \varphi_2)} \mathcal{F}\left[B(r, \theta_1, \theta_2)\right]_{(\varphi_1, \varphi_2)} dr \tag{24}$$

Using homogeneous transformation (D-H method) for CB_3 Taking into account that we are working on spherical coordinates, the frames of figure 2, where a new frame F'_W is established, which is equal to F_{A_2}, are the best election.

The idea is to perform the transformation of the workspace points related to F_W, to the ones related to F_{A_2} (F'_W) (see figure 2); that is, change from $(r, \varphi_1, \varphi_2)$ coordinates to $(r', \varphi'_1, \varphi'_2)$ coordinates.

Using the Denavit-Hartenberg method, we have $p' = {}^0_2\mathbf{T}^{-1} \cdot p$, and after the proper calculations we obtain the following expressions:

$$r' = \sqrt{l_2^2 + r^2 - 2rl_2(C\theta_1 C\theta_2 C\varphi_1 C\varphi_2 + S\theta_1 C\theta_2 S\varphi_1 C\varphi_2 + S\theta_2 S\varphi_2)}$$
$$\varphi'_1 = artg\left(\frac{-r(C\theta_1 S\theta_2 C\varphi_1 C\varphi_2 + S\theta_1 S\theta_2 S\varphi_1 C\varphi_2 - C\theta_2 S\varphi_2)}{r(C\theta_1 C\theta_2 C\varphi_1 C\varphi_2 + S\theta_1 C\theta_2 S\varphi_1 C\varphi_2 + S\theta_2 S\varphi_2) - l_2}\right)$$
$$\varphi'_2 = artg\left(\frac{z'}{\sqrt{x'^2 + y'^2}}\right)$$

with

$$
\begin{aligned}
x' &= r(C\theta_1 C\theta_2 C\varphi_1 C\varphi_2 + S\theta_1 C\theta_2 S\varphi_1 C\varphi_2 + S\theta_2 S\varphi_2) - l_2 \\
y' &= -r(C\theta_1 S\theta_2 C\varphi_1 C\varphi_2 + S\theta_1 S\theta_2 S\varphi_1 C\varphi_2 - C\theta_2 S\varphi_2) \\
z' &= r(S\theta_1 C\varphi_1 C\varphi_2 - C\theta_1 S\varphi_1 C\varphi_2)
\end{aligned}
$$

Furthermore, since the elbow is a revolution articulation with the turning axis parallel to that of the shoulder articulation (figure 2), within the sphere covered by $(r, \varphi_1, \varphi_2)$, only the obstacles within the disk of l_3 radius, i.e. the longitude of the third element, and φ'_1 angle in $[-\pi, \pi]$, can be obstacles for $\mathbf{A}_3$. In this situation, it is more efficient to transform only those points that are in the disk of interest, that is to say, with $\varphi_1 = \theta_1$ (related to original frame) and $r' < l_3$ (related to the transformed one). This concept is illustrated in figure 3.

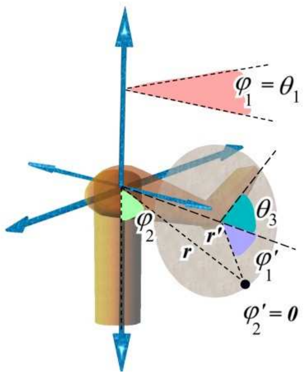

Fig. 3. Any point with $\varphi_1 = \theta_1$ pertains to the disk of interest

With the introduced frame change, instead of working in the 3D space, we work in the plane, and so, now the expression 18 can be evaluated as

$$CB_3(\theta_1, \theta_2, \theta_3) = \int A'_3(\theta_3, r', \varphi'_1) B'(r', \varphi'_1) dr' d\varphi'_1 \tag{25}$$

Use of Convolution for CB_3 calculation As it can be seen, since there is a relationship between θ_3 and φ'_1, expression 25 can be written as

$$CB_3(\theta_1, \theta_2, \theta_3) = \int A'_{3_{(0)}}(r', \varphi'_1 - \theta_3) B'(r', \varphi'_1) dr' d\varphi'_1 \tag{26}$$

that can be simplified, applying the convolution theorem, and obtain the final expression:

$$\mathcal{F}\left[CB_3(\theta_1, \theta_2, \theta_3)\right] = \int \mathcal{F}\left[\bar{A}'_{3_{(0)}}(r', \theta_3)\right]_{\varphi'_1} \mathcal{F}\left[B'(r', \theta_3)\right]_{\varphi'_1} dr' \tag{27}$$

It must be noted that, on the contrary to the previous expression, where it was necessary to perform bidimensional Fourier transforms, in this case the Fourier transforms are one-dimensional, since it is only necessary to *sweep* disks.

Finally, note that, in order to use the Deconstruction method, a discretization must be performed.

6 Conclusions

A fast and new general method for the evaluation of the configuration space of any kinematic chain was presented. The possibility of simplification of the C-space evaluating process by means of the application of a simple and repetitive operation for each link in the kinematic chain was shown. As case study, the proposed method was applied to a 3-DOFs arm, showing its potential for collision detection of articulated bodies such as human figures.

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